NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH, NATURAL RESOURCES AND APPLIED SCIENCES
SCHOOL OF NATURAL AND APPLIED SCIENCES
DEPARTMENT OF MATHEMATICS, STATISTICS AND ACTUARIAL SCIENCE

QUALIFICATION: Bachelor of Science Honours in Applied Mathematics	
QUALIFICATION CODE: 08BSHM	**LEVEL:** 8
COURSE CODE: ANA801S	**COURSE NAME:** APPLIED NUMERICAL ANALYSIS
SESSION: JULY 2023	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 120 (to be converted to 100%)

2ND OPPORTUNITY/SUPPLEMENTARY EXAMINATION QUESTION PAPER	
EXAMINERS	PROF S. A. REJU
MODERATOR:	PROF S. MOTSA

INSTRUCTIONS
1. Attempt ALL the questions. 2. All written work must be done in blue or black ink and sketches must be done in pencils. 3. Use of COMMA is not allowed as a DECIMAL POINT.

PERMISSIBLE MATERIALS

1. Non-programmable calculator without a cover.

THIS QUESTION PAPER CONSISTS OF 3 PAGES (including this front page)

QUESTION 1 [30 MARKS]

(a) Discuss the contrast between a quadrature rule and the adaptive rule. **[3]**

(b) Consider the integral **[27]**

$$\int_a^b f(x)dx = \int_1^3 e^{2x}sin(3x)dx$$

Using the Adaptive Simpson's Method and an error $\epsilon = 0.2$, obtain the approximate value of the above integral (for computational ease, using where appropriate the following as done in class):

$$\frac{1}{10}\left|S(a,b) - S(a,\frac{a+b}{2}) - S(\frac{a+b}{2},b)\right|$$

where

$$\int_a^b f(x)dx = (S(a,b) - \frac{h^5}{90}f^{(4)}(\xi),\ \ \xi\epsilon(a,b)$$

QUESTION 2 [30 MARKS]

Discuss exhaustively the Romberg Method Extrapolation process to show that the nth order extrapolation employed by the method is given by:

$$I_{Improved} = \frac{4^n I_{More-accurate} - I_{Less\ accurate}}{4^n - 1}$$

QUESTION 3 [30 MARKS]

(a) (i) State the Steepest Descent Algorithm **[6]**
(ii) State the theorem that guarantees that the Steepest Descent method ensures some progress in the direction of the minimum of the objective function during each iteration. **[4]**

(b) Using the Steepest Descent Method, obtain the minimum of the following function:

$$f(x,y) = 4x^2 - 4xy + 2y^2$$

[20]

QUESTION 4 [30 MARKS]

(a) Define the Picard Method for solving the following Initial Value Problem (IVP)

$$\frac{dy}{dt} = y'(t) = f\big(t, y(t)\big), y(t_0) = y_0$$

and hence derive the Picard Iteration algorithm **[13]**

(b) Using the Picard method, find the solution, correct to 3 decimal places, of the following 1st order IVP at x = 0.1

$$\frac{dy}{dx} = x + y^2 \ , \ y(0) = 1$$

with $\boldsymbol{x(0) = x_0 = 0}$ **[17]**

END OF QUESTION PAPER TOTAL MARKS = 120